INVITED PAPER *Special Section on Progress & Trend of Superconductor-based Computers*

Development of Quantum Annealer Using Josephson Parametric Oscillators

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SUMMARY A Josephson parametric oscillator (JPO) is an interesting system from the viewpoint of quantum optics because it has two stable self-oscillating states and can deterministically generate quantum cat states. A theoretical proposal has been made to operate a network of multiple JPOs as a quantum annealer, which can solve adiabatically combinatorial optimization problems at high speed. Proof-of-concept experiments have been actively conducted for application to quantum computations. This article provides a review of the mechanism of JPOs and their application as a quantum annealer.

key words: *quantum optics, Josephson parametric oscillator, quantum annealing, quantum computation, superconducting qubits*

1. Quantum Computation

In accordance with Moore's Law, which states that the number of transistors on a chip doubles every 18 months [1], the performance of general-purpose processors including CPUs has improved every year while their price and power consumption have decreased. The operating frequency and the single-thread processing performance have almost reached their limits mainly due to power consumption constraints. These constraints have led to the development of many-core processors, the speedup of which is also limited by the number of programs sequentially executed. Therefore, speedups have utilized specialized architectures in the right places, such as GPUs. Although GPUs cannot perform general-purpose processing like CPUs, they can perform massive parallel simple operations, which are quite useful for machine learning.

Quantum computers have attracted attention as a specialized architecture for their ability to solve problems that are difficult using conventional computers. In contrast to conventional computers, the information processing unit (bit) of which takes either two states, 0 or 1, a quantum computer is composed of quantum bits (qubits) that can take the superposition of 0 and 1 states. These computers can utilize the characteristic properties of quantum mechanics, such as the superposition of states, quantum tunneling, and quantum entanglement. Quantum computers can be roughly classified into two categories: a gate-based quantum computer [2] and a quantum annealer [3]. The gate-based quantum computer can calculate specific problems extremely quickly using the interference effect between the superposition of qubit states (2^n states for n qubits), and it is upwardly compatible with conventional computers. Quantum annealing is one of the metaheuristics of adiabatic quantum computation [3]–[6]. The quantum annealer encodes an Ising Hamiltonian on a network of qubits, and searches the global minima of the Hamiltonian by adiabatically changing the parameters of the Hamiltonian. Combinatorial optimization problems, whose cost functions can be encoded on the Ising Hamiltonian, have various economic and social applications including typical problems such as the traveling salesman problem and the knapsack problem. Since these problems become difficult to solve by classical computers as the problem size increases, the quantum annealer is expected to provide good solutions to these problems in a reasonable time.

Quantum computers have been developed using various kinds of physical implementations such as superconducting circuits, trapped ions, photons, nuclear magnetic resonance, and a nitrogen-vacancy center. Among them, superconducting circuits are one of the most intensively studied platforms partly because they are solid-state devices with good integration properties (fabrication technology developed for the semiconductor industry is applicable) and good quantum coherence due to their superconductivity [7]–[10]. The advantages may outweigh the disadvantage of having to be cooled down below 10 mK using a dilution refrigerator. We will mainly discuss quantum computers made of superconducting circuits in the following subsections.

The current milestone in the field of the gate-based quantum computer is the achievement of a noisy intermediate-scale quantum (NISQ) system [11], which is a machine containing from 50 to 100 qubits without error correction. A 53-qubit NISQ machine (developed by a group including Google) made of superconducting circuits enabled quantum supremacy for the first time in 2019 on an impractical computational process called random quantum circuit sampling [12]. Quantum supremacy is a milestone of quantum computation proposed by Preskill in 2012 [13], where a quantum computer is capable of performing a process that cannot be done in a reasonable amount of time using a conventional computer. Practical usages of gate-based quantum computers, such as cryptanalysis based on prime factorization of large composite numbers [14], require error correction of qubits. Quantum error correction has been hard to develop because it requires extremely low infidelity and a large number of physical qubits [15], [16].

In 2011, D-wave Systems of Canada released a 128-

Manuscript received July 28, 2021.
Manuscript revised October 6, 2021.
Manuscript publicized December 3, 2021.

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DOI: 10.1587/transele.2021SEP0006

qubit commercial quantum annealing machine based on a superconducting circuit for the first time [17]. The number of qubits in D-wave quantum annealers has steadily increased, and an annealer with more than 5000 qubits is now available [18]. Many applications have already been developed by D-wave user companies and researchers. In Japan, although no commercial quantum annealers have been developed [19], various annealers using classical or classical-quantum hybrid systems have been actively developed and commercialized. Simulated annealing utilizes conventional computer technologies such as FPGAs and GPUs, in which thermal-activation-type dynamics is simulated in calculations [20]–[22]. A coherent Ising machine utilizes optical phase states as qubits, and the interaction between qubits is achieved using conventional computation [23].

We are now performing research and development on a quantum annealer using superconducting parametric devices at NEC Corporation. In this paper, we introduce the historical background of superconducting parametric devices and quantum annealing, which NEC is striving to develop.

2. Superconducting Qubits and Readout

Figure 1 shows the circuit diagram of the first superconducting qubit demonstrated to work in 1999 by the NEC group [24]. The qubit was fabricated using electron beam lithography. Conduction electrons in a superconductor form pairs (Cooper pairs), which can move in and out of the box through tunnel junctions. The 0 and 1 states of the qubit can be defined as follows: while the 1 state has one extra Cooper pair in the box, the 0 state has no additional one. The two states can be read out by measuring the current flowing through the probe electrode. Although the coherence time was about 1 ns, the quantum superposition of these two states was observed.

Since then, superconducting qubit technology has made steady progress in performance indices such as coherence time and gate operation fidelity [8], [16]. The qubit readout is also an important element of the technology. Although various ways of realizing a qubit readout are possible, such as those using a single Josephson junction [25], a DC superconducting quantum interference device (dc-SQUID) [26], and a single-electron transistor [27], a method called dispersive readout [28], [29] has been most commonly used recently [30]. Figure 2 shows schematics of dispersive readout. A qubit, a two-level system composed of ground ($|g\rangle$) and excited ($|e\rangle$) states, is coupled to a linear resonator with a coupling strength of g. The system can be described using the Jaynes-Cummings model as follows [31],

$$\mathcal{H}_{\mathrm{JC}}/\hbar = \omega_{\mathrm{a}}\sigma^{\dagger}\sigma + \omega_{\mathrm{c}}a^{\dagger}a + g\left(\sigma^{\dagger}a + \sigma a^{\dagger}\right) \tag{1}$$

where $\omega_{\mathrm{a,c}}$ are the resonance frequencies of the qubit and the linear resonator, respectively, and σ, a are the annihilation operators of the qubit excitation and a mode of the linear resonator, respectively. This model had been originally proposed to describe the interaction between atoms and photons in a cavity, but later it was applied for the superconducting quantum circuits [28], [29]. When the detuning between the resonance frequencies of the qubit and the linear resonator is much larger than the coupling strength (dispersive limit, $|\Delta\omega| = |\omega_{\mathrm{a}} - \omega_{\mathrm{c}}| \gg g$), the mode of the linear resonator is partially mixed with the qubit states and turned into states having resonance frequencies $\omega_{\pm} = \omega_{\mathrm{c}} \pm \chi$ that depend on the qubit state, where $\chi \equiv g^2/|\omega_{\mathrm{c}} - \omega_{\mathrm{a}}|$ is the dispersive shift [28]. The state of the qubit can be read out by measuring the reflection coefficient at $\omega \sim \omega_{\mathrm{c}}$ thanks to the state dependence of the resonance frequency. However, the single-shot readout of the dispersive shift was impossible due to the low signal-to-noise ratio of the reflection coefficient, and the dispersive readout was performed by averaging the signals over many trials in the early days of the research of superconducting quantum circuits [32]. The low signal-to-noise ratio was partially due to the short coherence time of the qubit but also due to the high-noise temperature of the cryogenic amplifier used to amplify the signal (~5 K for typical high electron mobility transistor amplifiers). Therefore, several groups—including NEC—have tried to develop a microwave amplifier with lower noise tem-

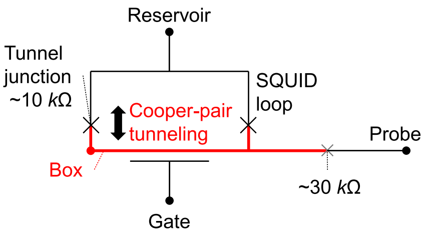

Fig. 1 Circuit diagram of the superconducting qubit demonstrated by NEC for the first time [24]. The Cooper-pair box is shown by the red solid line.

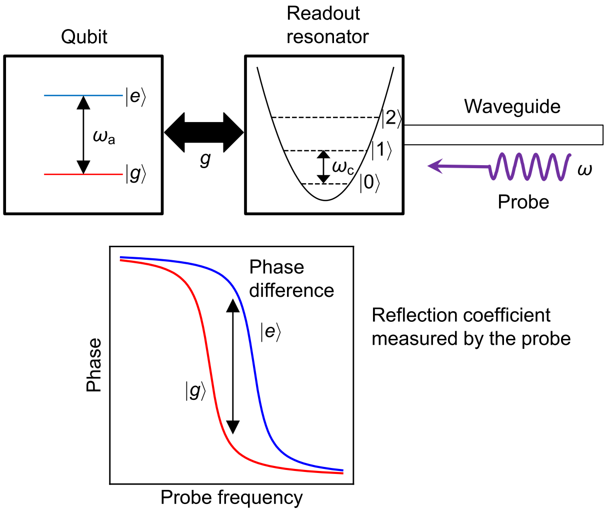

Fig. 2 Schematics of dispersive readout. A qubit (upper-left square) is coupled to a linear resonator (upper right square) with the coupling strength of g. The reflection coefficient of the linear resonator depends on the state of the qubit as shown in the lower graph.

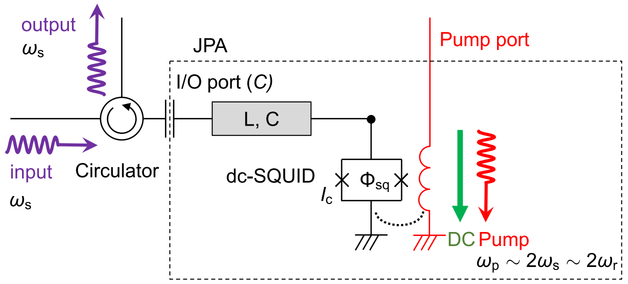

Fig. 3 Schematic of a Josephson Parametric Amplifier (JPA). A JPA shown by the dashed rectangle is a nonlinear microwave resonator with the resonance frequency ω_r, which is composed of linear LC circuit elements represented by the grey rectangle and a dc-SQUID. The JPA has two control ports, one of which is an I/O port capacitively coupled to an I/O line and the other is a pump port (the red line) inductively coupled (the dashed arc) to the SQUID. The I/O line is separated into input and output lines by a circulator. The purple arrows show the input and output microwaves with the signal frequency ω_s. The magnetic flux Φ_{sq} threading the SQUID can be controlled by applying DC currents (the green arrow) and pump microwaves (the red arrow) with the pump frequency of $\omega_p \sim 2\omega_s \sim 2\omega_r$ on the pump port.

perature using superconducting circuits, a Josephson parametric amplifier (JPA) [33]–[36].

Figure 3 shows a schematic of a reflection-type JPA. A JPA is an LC microwave resonator composed of linear cavity elements, such as a $\lambda/4$ coplanar waveguide, terminated with a dc superconducting quantum interferometer (dc-SQUID), which determines the boundary condition of the resonator. The JPA can be fabricated by forming a planar circuit on a chip with a superconducting thin film deposited on a highly resistive insulator substrate. The SQUID is formed separately by using the shadow evaporation technique. The dc-SQUID behaves as an inductor, where its effective inductance can be written as follows,

$$L_J = \frac{\Phi_0}{2\pi} \frac{1}{2I_c \left|\cos\left(\pi\Phi_{sq}/\Phi_0\right)\right|}, \tag{2}$$

where $\Phi_0 = h/(2e)$ is the flux quantum, I_c is the critical current of a Josephson junction, and Φ_{sq} is the magnetic flux threading the loop of the SQUID. We also assume a symmetrical dc-SQUID composed of Josephson junctions with identical critical currents for simplicity. Because the nonlinear inductance depends on the magnetic flux threading the loop of the SQUID, the resonance frequency of the JPA ω_r can be controlled by applying DC magnetic fluxes via a pump port inductively coupled to the SQUID.

When the resonance frequency of the JPA is modulated by AC pump microwaves at a frequency close to twice the resonance frequency (pump frequency, $\omega_p \sim 2\omega_r$), the JPA amplifies the input signal at $\omega_s \sim \omega_r$. When ω_s is exactly half the pump frequency, the amplifier is called to operate in a degenerate mode. The amplification is unique in that its gain depends on the relative phase between the signal and the pump. It is also practically important because it can beat the standard quantum limit in the noise added by amplifiers [37].

JPAs emerged in the 1960s [38]. Although the early stages of the development of JPAs faced a problem called noise rise [39], a noise temperature close to the quantum limit and squeezing of vacuum noise in the microwave regime were reported in 1990 [40]. Despite these achievements, JPAs had not been widely used before the 2000s partly because few applications were available. The development of the research field of superconducting quantum computers made JPAs an indispensable tool for readouts.

JPAs have been used as a preamplifier in a variety of experiments that require extremely high sensitivity. For example, they have been used to measure the position of micro mechanical oscillators with an accuracy exceeding the standard quantum limit [41] and to observe quantum jumps in superconducting qubits [42]. JPAs have been used not only as a low-noise amplifier but also as a squeezed microwave generator operating in the degenerate mode [37]. The squeezed microwaves generated by JPAs have been evaluated by a technique called quantum state tomography [43], and these microwaves were also used by our collaborators to demonstrate entanglement between continuous variable quantum states via two microwaves propagating along two spatially separated paths [44]. In parallel with the aforementioned experiments, the performance of the JPA itself continued to be improved. To enable simultaneous reading of multiple qubits using frequency multiplexing [45] in circuits with order of ten qubits, the JPA needs to have a wider bandwidth and higher gain saturation power. It would also be desirable that JPA has directivity in order to avoid the usage of bulky and magnetic circulators. Improved JPAs, such as impedance-matching JPAs [46], propagating-wave JPAs [47], [48], and coupled Josephson parametric converters [49], have been reported to achieve these goals.

3. Josephson Parametric Oscillator

The gain of the JPA generally increases as the pump amplitude increases, and the pump amplitude has a threshold proportional to the photon loss rate of the resonator [50]. When the pump amplitude exceeds the threshold, the JPA enters a self-oscillating state generating a signal at half the pump frequency [51]. This phenomenon is a type of parametric oscillation pumped by periodic variations in a parameter of the resonator, as shown in Fig. 4. Typically, the resonance frequency of a parametric oscillator is modulated at the twice of it. A nonlinear oscillator composed of Josephson junctions is called a Josephson parametric oscillator (JPO).

The JPOs take two self-oscillating states with an equal amplitude and well-defined phases of 0 or π. If there is no input signal, the two states occur completely randomly because they are degenerate. The degeneracy can be lifted by imposing input signals at the oscillation frequency (half the pump frequency), and the occurrence probabilities of the self-oscillating states can be controlled by the amplitude and phase of the input signals (phase locking), enabling JPOs to operate as phase detectors with high sensitivity [52], [53]. The same principle of operation, parametric oscillation, was

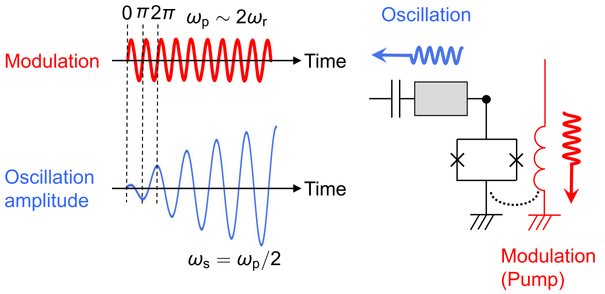

Fig. 4 Schematic of parametric oscillations. Right: a parametric oscillator is a resonator whose circuit parameter is periodically modulated. The resonance frequency is typically modulated at twice of it, $\omega_\mathrm{p} \sim 2\omega_\mathrm{r}$. A JPO can be driven by applying pump microwaves on the pump port (see Fig. 3). Left: when the modulation of the resonance frequency is larger than a threshold, the photon field builds up in the resonator and parametric oscillations with half the pump frequency ($\omega_\mathrm{s} = \omega_\mathrm{p}/2$) begin.

utilized with an LC nonlinear microwave resonator called a parametron, which was invented in the 1950s and used as a building block of early classical computers [54]. Moreover, optical parametric oscillators have been used as a light source with a variable frequency [55], [56].

The Hamiltonian of a JPO in a rotating frame at the oscillation frequency can be written as follows [57],

$$\mathcal{H}_\mathrm{jpo}/\hbar = \frac{K}{2}a^{\dagger 2}a^2 + \Delta a^\dagger a + \frac{p}{2}(a^{\dagger 2} + a^2) + h\left(e^{\mathrm{i}\theta}a^\dagger + e^{-\mathrm{i}\theta}a\right), \tag{3}$$

where K is the Kerr nonlinearity ($K = \omega_{12} - \omega_\mathrm{r} < 0$, ω_{12} is the transition frequency between the Fock states $|1\rangle$ and $|2\rangle$ of a JPO), $\Delta \equiv \omega_\mathrm{r} - \omega_\mathrm{s}$ is the detuning of the oscillation frequency from the resonance frequency, p is the pump amplitude, h and θ are the amplitude and phase of a phase-locking signal, respectively, and a is the annihilation operator of a mode of a JPO. When $|K|$ is larger than the photon loss rate and the energy-level separation is greater than the level widths, the quantum-mechanical effects become apparent. By assuming for simplicity $h = \Delta = 0$, the vacuum $|0\rangle$ and $|1\rangle$ states are the degenerate highest energy eigenstates in the rotating frame without the pump. The corresponding degenerate highest energy eigenstates under the pump are the coherent states $|\pm\alpha\rangle$, where the oscillation amplitude is $\alpha = \sqrt{p/|K|}$. The oscillation amplitude corresponds to the maxima of a metapotential $U(x) = \frac{K}{2}|x|^4 + \frac{p}{2}(x^{*2} + x^2)$, which can be obtained by replacing the operators in Eq. (3) with a c-number x. When the pump is adiabatically induced, the initial ground state $|0\rangle$ adiabatically evolves to the superposition of the coherent states (even cat state due to parity conservation) [58], [59].

4. Quantum Annealer Using JPOs

A theoretical proposal has been made to encode the Ising Hamiltonian on a network of coupled JPOs, which can be utilized for quantum adiabatic computation [57], [60]–[64]. In the proposals, JPOs are operated in the Kerr parametric oscillator (KPO) regime, where the nonlinearity of JPOs is larger than their photon loss rate (single-photon Kerr regime). The KPOs can be utilized for gate-type quantum computation [59], [65], [66], Boltzmann sampling [67] and studying quantum phase transitions [68], [69], and the KPOs have been experimentally studied [70]–[72].

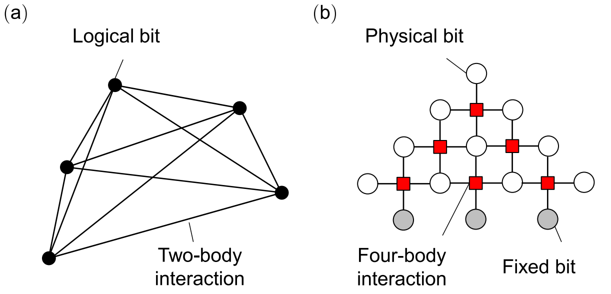

Fig. 5 Schematic of a graph structure of an Ising model. (a) an Ising model with an all-to-all connected graph, where the number of logical spins is five. (b) a scalable implementation of physical spins using the LHZ scheme, where the logical spins correspond to (a).

The self-oscillating states of KPO, $|\pm\alpha\rangle$, correspond to the Ising spin $s_i = \pm 1$, where the coupling between KPOs acts as the coupling between spins, and the phase-locking signal applied to each KPO acts as the local magnetic field. The initial state of the adiabatic computation is the vacuum state without a pump. By adiabatically applying pumps, the initial state adiabatically evolves to the ground state of the target Hamiltonian, which corresponds to the spin configuration minimizing the Ising Hamiltonian and can be measured via the oscillation phase. Therefore, combinatorial optimization problems encoded on the Ising Hamiltonian are expected to be solved using the network of KPOs.

The Ising Hamiltonian of combinatorial optimization problems generally requires all-to-all connectivity of a two-body interaction between logical bits as shown in Fig. 5 (a). However, constructing a scalable large all-to-all connected system is very difficult to achieve because the superconducting quantum circuits need to be fabricated on a plane if a high degree of coherence is required. The quantum annealer developed by D-wave utilizes a sparsely coupled network of qubits, such as Chimera and Pegasus graphs [73], [74], and embeds combinatorial optimization problems with all-to-all connected logical spins into the graphs of the physical spins [75]. The graph embedding requires complex settings of additional ancillary qubits, which may affect the success rate of the calculation [76]. The Lechner-Hauke-Zoller (LHZ) scheme, a method in which the physical bits are placed using simple rules in a scalable manner, has been proposed to improve the complexity of graph embedding [77]. This scheme consists of physical bits and a four-body interaction coupler arranged as shown in Fig. 5 (b). The physical bits of the LHZ scheme correspond to the two-spin product of the logical qubits ($s'_{ij} = s_i s_j$, where s' represents physical qubits). Although the number of physical bits increases to $N(N-1)/2$ for N logical bits, the two-body interaction between the logical bits is replaced by the local magnetic field applied to each physical bits, which is easy for physical implementation. Because of the redundancy in the number of bits, the physical bits can take configurations that cannot be achieved by the logical bits. For example, the configuration $s'_{01} = 1$, $s'_{12} = 1$, $s'_{20} = -1$ does not correspond to any configurations of the logical bits s_0, s_1 and s_2. By incorporating four-body interactions as shown in Fig. 5 (b), the energies of spin configurations inconsistent with logical bits are heightened and then effectively excluded from the computational space. It is an efficient method for scalable systems because the scheme is easy to be physically implemented on a plane and because the relationship between physical and logical bits is simple. NEC is currently conducting research and development of an annealing quantum computer using JPOs as part of the NEDO project.

5. Conclusion

This paper provided an overview of superconducting qubits and readouts, JPOs, and quantum annealers using JPOs. A JPO is a device used as a qubit in the quantum annealing system being developed by the NEC group.

In addition to the qubits, which are the heart of the device, the mounting and wiring technology for connecting the control unit to the many qubits within a dilution refrigerator is equally important. We are working with the National Institute of Advanced Industrial Science and Technology and others on this wiring technology in the NEDO project.

Acknowledgments

This work was partly supported by JST ERATO (Grant No. JPMJER1601). This paper is based on results obtained from a project, JPNP16007, commissioned by the New Energy and Industrial Technology Development Organization (NEDO).

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