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PAPER Special Section on Information Theory and Its Applications

Detection Probability of Poor Responses in Questionnaires with Quality Control Questions*

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SUMMARY In the realm of web-based surveys, ensuring the quality of responses is a crucial yet challenging task. This study addresses the issue of detecting poor responses, particularly focusing on the phenomenon of 'satisficing' - a situation where respondents provide minimal effort responses. Traditional methods such as the Instructional Manipulation Check (IMC) and Directed Question Scale (DQS) have been commonly employed to tackle this issue. However, their effectiveness is often limited due to various constraints. This paper introduces a theoretical framework and a generalized model for the design of questionnaires. This framework aims to improve the detection of poor responses, thus enhancing the reliability and validity of survey data. Through numerical experiments, the paper demonstrates the applicability and effectiveness of the proposed model. The study's approach is based on a thorough analysis of response patterns and the integration of quality control questions within the survey structure. The findings of this research have significant implications for the field of survey methodology, providing a more robust and systematic way of ensuring data integrity in web-based surveys.

key words: *Satisficing in Surveys, Quality Control Questions, Instructional Manipulation Check, Directed Question Scale*

1. Introduction

In an era where data-driven decision-making has become a cornerstone across various sectors, surveys have emerged as pivotal tools for gathering insights and opinions. The transition from traditional paper-based methods to web-based surveys has revolutionized the way organizations, from academic researchers to market analysts, collect data. This digital shift, while offering unprecedented reach and efficiency, has introduced new challenges in ensuring data integrity and representativeness.

Web-based surveys, despite their popularity, are susceptible to a range of biases that compromise data reliability. Among these, the phenomenon of "satisficing"[1] – a term coined to describe the behavior where respondents provide satisfactory but suboptimal answers – has become increasingly prevalent. This behavior is more than just a minor inconvenience; it poses a significant threat to the quality of data collected. Satisficing manifests in various forms, from respondents skimming through questions without due attention to providing arbitrary answers to speed through the survey. Such responses not only dilute the accuracy of the survey results but can also lead to misleading conclusions, particularly in scenarios where precise data is crucial for policy-making or business strategies.

Traditionally, to detect poor responses caused by Satisficing, methods involving the embedding of special questions known as quality control questions have been considered. These quality control questions include methods such as the Instructional Manipulation Check (IMC)[2], the Directed Question Scale (DQS)[3], and Consistency Check Questions (CCQs). The IMC is a method that includes specific instructions in the question to ensure the respondent is reading the question correctly. For example, it might instruct, "Please always choose option 1 for this question." The DQS includes specific instructions among the choices. For instance, in a Likert scale item, it might include instructions like, "Select the far left option for this item." CCQs are used to verify that a participant's responses are consistent throughout the survey. This might involve asking similar questions in different formats several times. A lot of research has been conducted on the extent to which actual surveys are improved by using these quality control questions [3]–[11]. However, these methods do not guarantee the detection of all poor responses. Also, the theoretical basis for the detection capabilities of quality control questions has not yet been fully elucidated.

Therefore, our research aims to establish a theoretical framework for questionnaire design to detect poor responses. We propose a generalized model for this purpose, derive theoretical probabilities of detecting poor responses, and provide guidelines for effective survey design through numerical experiments. This approach seeks to enhance the reliability of web-based surveys by addressing the critical issue of response quality.

2. Definition of Questionnaire Design Model

2.1 Design of Questions

In this paper, we define questions related to the information desired in a survey as "information questions". Additionally, we define dummy questions added for detecting poor responses as "quality control questions". We assume both information and quality control questions have a multiple-choice format with D options, represented by $\{0, 1, \ldots, D-1\}$. L quality control questions are concatenated to K information questions. The response sequence of an arbitrary respondent to the information questions is denoted as $\boldsymbol{x} = (x_1, x_2, \ldots, x_K)$. Similarly, the re-

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Table 1 QCQ Function

x_k	$f_l(x_k)$
0	$h_{l,0}$
1	$h_{l,1}$
$\cdots$	$\cdots$
$D-1$	$h_{l,D-1}$

sponse sequence to the quality control questions is denoted as $\boldsymbol{c} = (c_1, c_2, \ldots, c_L)$.

The l-th quality control question is defined as a question for which the response is uniquely determined according to the function $f_l(\tilde{\boldsymbol{x}})$ for any response $\tilde{\boldsymbol{x}} = (\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_K)$ from a suitable respondent who is not a poor responder. This function $f_l(\tilde{\boldsymbol{x}})$ is referred to as the "quality control question (QCQ) function", and its determination constitutes the method for designing quality control questions. In the following, we focus on a class of QCQ functions where the response to the l-th quality control question depends solely on a specific k-th response. Therefore we represent the QCQ function as $f_l(\tilde{\boldsymbol{x}}_k)$.

We present a general representation of the input and output of a certain QCQ function in Table 1. That is, $f_l(x_k) = h_{l,x_k}$. There are D^D patterns of QCQ functions. For $D = 3$, there exist 27 patterns as shown in Table 2. For example, the QCQ function of pattern p1 ($f_l(0) = 0, f_l(1) = 0, f_l(2) = 0$) represents a quality control question that always results in a 0, regardless of the content of the information questions. This type of quality control question can be considered as used in IMC or DQS. Another example, the QCQ function of pattern p16 ($f_l(0) = 1, f_l(1) = 2, f_l(2) = 0$), represents a quality control question that queries similar content to the information questions but with a different order of options. The set of these QCQ functions, when combined as L functions, is defined as $\mathcal{F} = \{f_1, f_2, \ldots, f_L\}$.

We detect poor responses by using defined quality control questions. For a respondent's response sequences $\boldsymbol{x}, \boldsymbol{c}$ and the set of quality control questions $\mathcal{F}$, a response is identified as poor if there exists a pair of x_k and c_l that satisfies the following condition:

$$f_l(x_k) \neq c_l, \quad l = 1, 2, \ldots, L. \tag{1}$$

In other words, if the predicted response to a quality control question $f_l(x_k)$ differs from the actual response c_l, the response is considered poor. An example of detecting poor responses is shown in Fig.1.

2.2 Poor Response Model

Different patterns of response methods by poor responders can be considered. We model the scenario where respondents answer probabilistically without thoroughly reading the questions. Therefore, it is assumed that a poor responder selects the options for each question independently, with a constant probability, regardless of the question content. This probability is referred to as the "response probability" and is defined as follows.

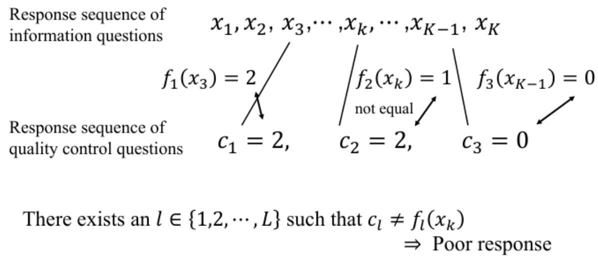

Fig. 1 Example of detecting poor responses

$$\Pr\{x_k = i\} = \theta_i,$$
$$i = 0, 1, \ldots, D-1, \quad k = 1, 2, \ldots, K. \tag{2}$$

$$\Pr\{c_l = i\} = \theta_i$$
$$i = 0, 1, \ldots, D-1, \quad l = 1, 2, \ldots, L. \tag{3}$$

$$\sum_{i=0}^{D-1} \theta_i = 1. \tag{4}$$

$$\boldsymbol{\theta} = (\theta_0, \theta_1, \ldots, \theta_{D-1}). \tag{5}$$

Furthermore, let us represent the probability density function of a poor respondent with a response probability of $\boldsymbol{\theta}$ as $p(\boldsymbol{\theta})$. It is assumed that all poor respondents are independently assigned a response probability $\boldsymbol{\theta}$ following $p(\boldsymbol{\theta})$.

As a concrete example of $p(\boldsymbol{\theta})$, consider a Dirichlet distribution with parameters $\boldsymbol{\alpha} = (\alpha_0, \alpha_1, \ldots, \alpha_{D-1})$, as represented by the following equation:

$$p(\boldsymbol{\theta}) = \frac{\Gamma(\sum_{i=0}^{D-1} \alpha_i)}{\Gamma(\alpha_0) \ldots \Gamma(\alpha_{D-1})} \theta_0^{\alpha_0 - 1} \theta_1^{\alpha_1 - 1} \ldots \theta_{D-1}^{\alpha_{D-1} - 1}, \tag{6}$$

where $\Gamma(\cdot)$ is Gammma function. In cases where $p(\boldsymbol{\theta})$ follows a Dirichlet distribution, we will refer to this as "poor respondents following the Dirichlet distribution model".

3. Average Poor Response Detection Probability

3.1 Definition of Average Poor Response Detection Probability

Let $R(\boldsymbol{\theta}, \mathcal{F})$ denote the probability that a response from a poor respondent with response probability $\boldsymbol{\theta}$ is detected as a poor response. $R(\boldsymbol{\theta}, \mathcal{F})$ is defined as follows:

$$R(\boldsymbol{\theta}, \mathcal{F}) = Pr\{\boldsymbol{x}, \boldsymbol{c} | f_l(x_k) \neq c_l, {}^{\exists} f_l \in \mathcal{F}\}. \tag{7}$$

$R(\boldsymbol{\theta}, \mathcal{F})$ represents the probability that a response from an individual poor respondent is detected as poor through the

Table 2 All Patterns of QCQ Functions for $D = 3$

x_k	$f_l(x_k)$																										
	p1	p2	p3	p4	p5	p6	p7	p8	p9	p10	p11	p12	p13	p14	p15	p16	p17	p18	p19	p20	p21	p22	p23	p24	p25	p26	p27
0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2
1	0	0	0	1	1	1	2	2	2	0	0	0	1	1	1	2	2	2	0	0	0	1	1	1	2	2	2
2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2

quality control questions $\mathcal{F}$. When analyzing questionnaires, considering the inclusion of faulty responses from multiple respondents, it becomes more interesting to know what proportion of poor responses can be detected among all faulty respondents.

Therefore, the average poor response detection probability $Q(\mathcal{F})$ is defined by the following equation:

$$Q(\mathcal{F}) = E[R(\boldsymbol{\theta}, \mathcal{F})], \\ = \int_{\boldsymbol{\theta}} R(\boldsymbol{\theta}, \mathcal{F}) p(\boldsymbol{\theta}) d\boldsymbol{\theta}. \quad (8)$$

$Q(\mathcal{F})$ does not represent the probability of detecting a response as poor among all survey respondents. Instead, it represents the probability of detecting a response as poor among all poor respondents.

3.2 Average Poor Response Detection Probability for a General Poor Response Model

The average poor response detection probability can be determined by the following theorem:

Theorem 1. *Let us assume the l-th QCQ function is represented as in Table 1, and all QCQ functions use different information questions as their arguments. When the probability of a poor respondent appearing with response probability $\boldsymbol{\theta}$ follows $p(\boldsymbol{\theta})$, the average poor response detection probability can be calculated by the following equation:*

$$Q(\mathcal{F}) = 1 - \prod_{l=1}^{L} \sum_{d=0}^{D-1} E[\theta_d \theta_{h_{l,d}}], \quad (9)$$

where, $E[\theta_d \theta_{h_{l,d}}]$ is defined by the following equation:

$$E[\theta_d \theta_{h_{l,d}}] = \int \theta_d \theta_{h_{l,d}} p(\boldsymbol{\theta}) d\boldsymbol{\theta}. \quad (10)$$

Proof. Firstly, define the probability of detecting a poor response using the l-th quality control question as $R_l(\boldsymbol{\theta}, f_l)$, which is defined below:

$$R_l(\boldsymbol{\theta}, f_l) = Pr\{\boldsymbol{x}, \boldsymbol{c} | f_l(x_k) \neq c_l\}. \quad (11)$$

For all QCQ functions f_l included in $\mathcal{F}$, when different x_k are used as arguments, the following holds:

$$R(\boldsymbol{\theta}, \mathcal{F}) = 1 - \prod_{l=1}^{L} (1 - R_l(\boldsymbol{\theta}, f_l)). \quad (12)$$

Here, $Q(\mathcal{F})$ can be expanded as follows:

$$Q(\mathcal{F}) = E[R(\boldsymbol{\theta}, \mathcal{F})], \\ = E[1 - \prod_{l=1}^{L} (1 - R_l(\boldsymbol{\theta}, f_l))], \\ = 1 - E[\prod_{l=1}^{L} (1 - R_l(\boldsymbol{\theta}, f_l))], \\ = 1 - \prod_{l=1}^{L} E[1 - R_l(\boldsymbol{\theta}, f_l)], \\ = 1 - \prod_{l=1}^{L} (1 - E[R_l(\boldsymbol{\theta}, f_l)]). \quad (13)$$

Furthermore, when a certain l-th quality control question uses the QCQ function from Table 1, $R_l(\boldsymbol{\theta}, f_l)$ can be determined as follows:

$$R_l(\boldsymbol{\theta}, f_l) = 1 - \{\theta_0 \theta_{h_{l,0}} + \theta_1 \theta_{h_{l,1}} + \ldots + \theta_{D-1} \theta_{h_{l,d-1}}\}. \quad (14)$$

Therefore, taking the expected values of both sides, the following equation holds:

$$E[R_l(\boldsymbol{\theta}, f_l)] = E[1 - \sum_{d=0}^{D-1} \theta_d \theta_{h_{l,d}}], \\ = 1 - \sum_{d=0}^{D-1} E[\theta_d \theta_{h_{l,d}}]. \quad (15)$$

Hence, by substituting equation (15) into equation (13), the theorem holds. □

3.3 Average Detection Probability of Poor Responses in the Dirichlet Distribution Model

As previously mentioned, the average detection probability of poor responses is determined by equation (9). However, in a general $p(\boldsymbol{\theta})$, it is not always possible to analytically determine $E[\theta_d \theta_{h_d}]$. Therefore, we calculate the value of $E[\theta_d \theta_{h_d}]$ for the case where the poor responders follow a Dirichlet distribution model.

Theorem 2. *When the poor responses follow a Dirichlet distribution model, the average poor response detection probability can be calculated by the following equation:*

$$Q(\mathcal{F}) = 1 - \frac{\prod_{l=1}^{L} \sum_{d=0}^{D-1} \alpha_d (\alpha_{h_{l,d}} + I_d(h_{l,d}))}{(\sum_{i=0}^{D-1} \alpha_i)^L (\sum_{i=0}^{D-1} \alpha_i + 1)^L}, \quad (16)$$

where, $I_d(a)$ is defined by the following equation:

$$I_d(a) = \begin{cases} 1 & , d = a, \\ 0 & , otherwise. \end{cases} \tag{17}$$

Proof. From the definition of the Dirichlet distribution, the following equation holds:

$$\begin{aligned} &E[\theta_d \theta_{h_d}] \\ &= \frac{\Gamma(\sum_{i=0}^{D-1} \alpha_i)}{\Gamma(\alpha_0)\cdots\Gamma(\alpha_{D-1})} \\ &\quad \times \int \theta_d \theta_{h_d} \theta_0^{\alpha_0 - 1} \theta_1^{\alpha_1 - 1} \ldots \theta_{D-1}^{\alpha_{D-1}-1} d\boldsymbol{\theta}. \end{aligned} \tag{18}$$

Now, when $h_d = d$, the following equation holds:

$$\begin{aligned} &\int \theta_d \theta_{h_d} \theta_0^{\alpha_0 - 1} \theta_1^{\alpha_1 - 1} \cdots \theta_{D-1}^{\alpha_{D-1}-1} d\boldsymbol{\theta} \\ &= \int \theta_d^{\alpha_d + 1} \prod_{j \neq d} \theta_j^{\alpha_j - 1} d\boldsymbol{\theta}, \\ &= \frac{\Gamma(\alpha_d + 2) \prod_{j\neq d} \Gamma(\alpha_j)}{\Gamma(\sum_{i=0}^{D-1} \alpha_i + 2)}, \\ &= \frac{\alpha_d(\alpha_d + 1)\Gamma(\alpha_0)\ldots\Gamma(\alpha_{D-1})}{(\sum_{i=0}^{D-1} \alpha_i)(\sum_{i=0}^{D-1} \alpha_i + 1)\Gamma(\sum_{i=0}^{D-1} \alpha_i)}. \end{aligned} \tag{19}$$

Therefore, the following holds:

$$E[\theta_d \theta_{h_d}] = \frac{\alpha_d(\alpha_d + 1)}{(\sum_{i=0}^{D-1} \alpha_i)(\sum_{i=0}^{D-1} \alpha_i + 1)}. \tag{20}$$

Moreover, when $h_d \neq d$, the following equation holds:

$$\begin{aligned} &\int \theta_d \theta_{h_d} \theta_0^{\alpha_0 - 1} \theta_1^{\alpha_1 - 1} \ldots \theta_{D-1}^{\alpha_{D-1}-1} d\boldsymbol{\theta} \\ &= \int \theta_d^{\alpha_d} \theta_{h_d}^{\alpha_{h_d}} \prod_{j \neq d, h_d} \theta_j^{\alpha_j - 1} d\boldsymbol{\theta}, \\ &= \frac{\Gamma(\alpha_d + 1)\Gamma(\alpha_{h_d} + 1) \prod_{j\neq d, h_d} \Gamma(\alpha_j)}{\Gamma(\sum_{i=0}^{D-1} \alpha_i + 2)}, \\ &= \frac{\alpha_d \alpha_{h_d} \Gamma(\alpha_0)\ldots\Gamma(\alpha_{D-1})}{(\sum_{i=0}^{D-1} \alpha_i)(\sum_{i=0}^{D-1} \alpha_i + 1)\Gamma(\sum_{i=0}^{D-1} \alpha_i)}. \end{aligned} \tag{21}$$

Therefore, the following holds:

$$E[\theta_d \theta_{h_d}] = \frac{\alpha_d \alpha_{h_d}}{(\sum_{i=0}^{D-1} \alpha_i)(\sum_{i=0}^{D-1} \alpha_i + 1)}. \tag{22}$$

The theorem is obtained by substituting equations (20) and (22) into equation (9). □

Consequently, if the response probability $\boldsymbol{\theta}$ of the poor responders follows the Dirichlet distribution, the average detection probability of poor respondents $Q(\mathcal{F})$ can be determined from the parameters of the Dirichlet distribution $\boldsymbol{\alpha}$ and the QCQ functions. By using this average detection probability, efficient QCQ functions can be selected, allowing surveys to be designed to detect poor responses.

4. Numerical Analysis

To specifically determine the average detection probability of poor responses when they follow the Dirichlet distribution model, we vary the parameters of the Dirichlet distribution $\boldsymbol{\alpha}$ and calculate the value of $Q(\mathcal{F})$ for each QCQ function.

For a Dirichlet distribution with $D = 3$, the parameters $\boldsymbol{\alpha} = (1, 1, 1)$ represent a uniform distribution. In other words, we consider a scenario where the response probability of a certain poor respondent is probabilistically determined from a uniform distribution. Additionally, when each parameter in $\boldsymbol{\alpha}$ is greater than 1, the Dirichlet distribution becomes unimodal, and the larger the parameter values, the more the distribution concentrates around a single point. Particularly, when $\alpha_0 = \alpha_1 = \alpha_2$, the distribution becomes unimodal centered around $\theta_0 = 1/3, \theta_1 = 1/3, \theta_2 = 1/3$. Therefore, when $\alpha_0 = \alpha_1 = \alpha_2 = 100$, the Dirichlet distribution forms a sharp peak around $\theta_0 = 1/3, \theta_1 = 1/3, \theta_2 = 1/3$. This implies that in this case, each faulty respondent is approximately equally likely to give a poor response with probabilities $\theta_0 = 1/3, \theta_1 = 1/3, \theta_2 = 1/3$.

5. Discussion

As can be seen from Table 3, the average detection probability of poor responses varies with different patterns of quality control questions based on the parameters $\boldsymbol{\alpha}$. If the distribution of response probabilities of the poor responders is known in advance, efficient quality control question design is possible using the average detection probabilities derived in this study.

However, in general, the distribution of response probabilities of poor responders is unknown. For example, as shown in Section 2, p1 is a QCQ function that represents IMC or DQS. In this case, as shown in Table 3, it can be seen that there is a large variation in the detection probability depending on the value of $\boldsymbol{\alpha}$. Especially when there are many poor responders who are more likely to answer the first question, such as in cases where $\boldsymbol{\alpha} = (10, 1, 1)$ or $\boldsymbol{\alpha} = (100, 1, 1)$, the detection probability becomes extremely small. Therefore, while IMC and DQS are useful for certain $\boldsymbol{\alpha}$ values, their performance can become very low depending on $\boldsymbol{\alpha}$.

In cases where the distribution of response probabilities of the poor responders is unknown, selection of quality control question patterns based on the minimax criterion can be considered. That is, for each QCQ function, we choose the one with the highest average detection probability under the worst-case scenario of $\boldsymbol{\alpha}$. The values representing the smallest average detection probabilities for each quality control question pattern against the experimented $\boldsymbol{\alpha}$ are shown in the far right column of Table 3. From this, it is evident that the quality control question patterns p16 and p20 show the largest average detection probabilities against the most disadvantageous $\boldsymbol{\alpha}$. Patterns p16 and p20 rearrange the responses so that they do not overlap with the positions of the responses and options in the original information question $x_k = (0, 1, 2)$. For example, when there is a tendency for many respondents to select the first option more frequently, such as in the case of $\boldsymbol{\alpha} = (100, 1, 1)$, if the first option is

Table 3 Detection Probability of Poor Responses

α_0	0.5	0.5	0.5	1	10	1	1	10	10	1	10	100	1	1	100	100	1	100	
α_1	0.5	0.5	1	1	1	10	1	10	1	10	10	1	100	1	100	1	100	100	
α_2	0.5	1	1	1	1	1	10	1	10	10	10	1	1	100	1	100	100	100	min
p1	0.667	0.750	0.800	0.667	0.167	0.917	0.917	0.524	0.524	0.952	0.667	0.020	0.990	0.990	0.502	0.502	0.995	0.667	0.020
p2	0.667	0.750	0.743	0.667	0.224	0.859	0.917	0.524	0.719	0.758	0.667	0.029	0.981	0.990	0.502	0.746	0.751	0.667	0.029
p3	0.533	0.500	0.629	0.583	0.218	0.910	0.276	0.541	0.502	0.736	0.656	0.029	0.990	0.038	0.505	0.500	0.749	0.666	0.029
p4	0.533	0.667	0.629	0.583	0.218	0.276	0.910	0.502	0.541	0.736	0.656	0.029	0.038	0.990	0.500	0.505	0.749	0.666	0.029
p5	0.533	0.667	0.571	0.583	0.276	0.218	0.910	0.502	0.736	0.541	0.656	0.038	0.029	0.990	0.500	0.749	0.505	0.666	0.029
p6	0.400	0.417	0.457	0.500	0.269	0.269	0.269	0.519	0.519	0.519	0.645	0.038	0.038	0.038	0.502	0.502	0.502	0.664	0.038
p7	0.667	0.708	0.743	0.667	0.224	0.917	0.859	0.719	0.524	0.758	0.667	0.029	0.990	0.981	0.746	0.502	0.751	0.667	0.029
p8	0.667	0.708	0.686	0.667	0.282	0.859	0.859	0.719	0.719	0.563	0.667	0.038	0.981	0.981	0.746	0.746	0.507	0.667	0.038
p9	0.533	0.458	0.571	0.583	0.276	0.910	0.218	0.736	0.502	0.541	0.656	0.038	0.990	0.029	0.749	0.500	0.505	0.666	0.029
p10	0.800	0.833	0.829	0.750	0.808	0.865	0.923	0.545	0.740	0.935	0.677	0.971	0.981	0.990	0.505	0.749	0.993	0.668	0.505
p11	0.800	0.833	0.771	0.750	0.865	0.808	0.923	0.545	0.935	0.740	0.677	0.981	0.971	0.990	0.505	0.993	0.749	0.668	0.505
p12	0.667	0.583	0.657	0.667	0.859	0.859	0.282	0.563	0.719	0.719	0.667	0.981	0.981	0.038	0.507	0.746	0.746	0.667	0.038
p13	0.667	0.750	0.657	0.667	0.859	0.224	0.917	0.524	0.758	0.719	0.667	0.981	0.029	0.990	0.502	0.751	0.746	0.667	0.029
p14	0.667	0.750	0.600	0.667	0.917	0.167	0.917	0.524	0.952	0.524	0.667	0.990	0.020	0.990	0.502	0.995	0.502	0.667	0.020
p15	0.533	0.500	0.486	0.583	0.910	0.218	0.276	0.541	0.736	0.502	0.656	0.990	0.029	0.038	0.505	0.749	0.500	0.666	0.029
p16	0.800	0.792	0.771	0.750	0.865	0.865	0.865	0.740	0.740	0.740	0.677	0.981	0.981	0.981	0.749	0.749	0.749	0.668	0.668
p17	0.800	0.792	0.714	0.750	0.923	0.808	0.865	0.740	0.935	0.545	0.677	0.990	0.971	0.981	0.749	0.993	0.505	0.668	0.505
p18	0.667	0.542	0.600	0.667	0.917	0.859	0.224	0.758	0.719	0.524	0.667	0.990	0.981	0.029	0.751	0.746	0.502	0.667	0.029
p19	0.800	0.792	0.829	0.750	0.808	0.923	0.865	0.740	0.545	0.935	0.677	0.971	0.990	0.981	0.749	0.505	0.993	0.668	0.505
p20	0.800	0.792	0.771	0.750	0.865	0.865	0.865	0.740	0.740	0.740	0.677	0.981	0.981	0.981	0.749	0.749	0.749	0.668	0.668
p21	0.667	0.542	0.657	0.667	0.859	0.917	0.224	0.758	0.524	0.719	0.667	0.981	0.990	0.029	0.751	0.502	0.746	0.667	0.029
p22	0.667	0.708	0.657	0.667	0.859	0.282	0.859	0.719	0.563	0.719	0.667	0.981	0.038	0.981	0.746	0.507	0.746	0.667	0.038
p23	0.667	0.708	0.600	0.667	0.917	0.224	0.859	0.719	0.758	0.524	0.667	0.990	0.029	0.981	0.746	0.751	0.502	0.667	0.029
p24	0.533	0.458	0.486	0.583	0.910	0.276	0.218	0.736	0.541	0.502	0.656	0.990	0.038	0.029	0.749	0.505	0.500	0.666	0.029
p25	0.800	0.750	0.771	0.750	0.865	0.923	0.808	0.935	0.545	0.740	0.677	0.981	0.990	0.971	0.993	0.505	0.749	0.668	0.505
p26	0.800	0.750	0.714	0.750	0.923	0.865	0.808	0.935	0.740	0.545	0.677	0.990	0.981	0.971	0.993	0.749	0.505	0.668	0.505
p27	0.667	0.500	0.600	0.667	0.917	0.917	0.167	0.952	0.524	0.524	0.667	0.990	0.990	0.020	0.995	0.502	0.502	0.667	0.020

the same "0" as in x_k, like in p8, detection misses are more likely to occur. In contrast, p16 and p20 do not have the same response in the same position as x_k, which reduces detection misses. A specific construction method could be, for instance, posing a question with the same content as the k-th information question but altering the order of the options presented.

Furthermore, the above calculations are based on the scenario of having only one quality control question. If, as mentioned earlier, we add quality control questions like p16, p20 for different information questions against the worst $\boldsymbol{\alpha}$, it is understood from equation (13) that the detection probability increases, as shown in Fig. 2. When designing a survey, if it is possible to estimate the tolerable rate of poor responses, the necessary number of quality control questions can be designed accordingly.

Now, in order to focus on the parameter $\boldsymbol{\alpha}$ of the Dirichlet distribution, we will denote the second term on the right-hand side of Eq. (16) as $U_{\boldsymbol{\alpha}}(\mathcal{F})$. That is, we define it as follows:

$$U_\alpha(\mathcal{F}) = \frac{\prod_{l=1}^{L} \sum_{d=0}^{D-1} \alpha_d(\alpha_{h_{l,d}} + I_d(h_{l,d}))}{(\sum_{i=0}^{D-1} \alpha_i)^L (\sum_{i=0}^{D-1} \alpha_i + 1)^L}. \tag{23}$$

Also, we rewrite $Q(\mathcal{F})$ in Eq. (16) as $Q_\alpha(\mathcal{F})$. Therefore, $Q_\alpha(\mathcal{F}) = 1 - U_\alpha(\mathcal{F})$. We denote the infimum of $Q_\alpha(\mathcal{F})$ over $\boldsymbol{\alpha} \geq \mathbf{0}$ as $Q_{\inf}(\mathcal{F})$. That is, we define $Q_{\inf}(\mathcal{F})$ as follows:

$$Q_{\inf}(\mathcal{F}) = \inf_{\alpha \geq \mathbf{0}} Q_\alpha(\mathcal{F}) = 1 - \sup_{\alpha \geq \mathbf{0}} U_\alpha(\mathcal{F}). \tag{24}$$

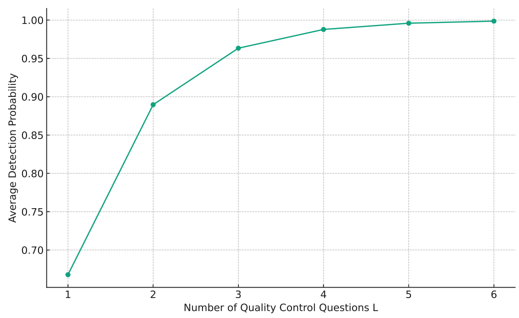

Fig. 2 Average Detection Probability of Poor Responses When Increasing the Number of quality control questions L

From this definition, it is clear that the following inequality holds for any value of the parameter $\boldsymbol{\alpha}$ of the Dirichlet distribution:

$$Q_\alpha(\mathcal{F}) \geq Q_{\inf}(\mathcal{F}). \tag{25}$$

The following lemma holds for the upper bound of $Q_{\inf}(\mathcal{F})$.

Lemma 1. For any $\mathcal{F}$, the following inequality holds:

$$Q_{\inf}(\mathcal{F}) \leq 1 - \frac{1}{D^L}. \tag{26}$$

Proof. For the proof, we assume $\boldsymbol{\alpha}$ that satisfies $\alpha_d = \frac{C}{D}, d = 0, 1, \ldots, D-1$, for a constant $C > 0$. Note that Eq. (25) holds for such specific C and $\boldsymbol{\alpha}$. Now, for such $\boldsymbol{\alpha}$ and any $\mathcal{F}$, the following equation holds:

$$U_\alpha(\mathcal{F}) = \frac{\prod_{l=1}^{L}\sum_{d=0}^{D-1}\frac{C}{D}(\frac{C}{D}+I_d(h_{l,d}))}{(\sum_{i=0}^{D-1}\frac{C}{D})^L(\sum_{i=0}^{D-1}\frac{C}{D}+1)^L} \tag{27}$$

$$= \frac{\prod_{l=1}^{L}\left(\frac{C}{D}+\frac{1}{D}\sum_{i=0}^{D-1}I_d(h_{l,d})\right)}{(C+1)^L}. \tag{28}$$

Dividing the numerator and denominator by C^L and taking the limit as $C \to \infty$, we obtain:

$$\lim_{C\to\infty} U_\alpha(\mathcal{F}) = \frac{1}{D^L}. \tag{29}$$

Thus, the lemma holds. □

Next, we consider evaluating the lower bound of $Q_{\inf}(\mathcal{F})$. We define $n_{d,j}$ as the number of functions among the L functions $f_1, f_2, \ldots, f_L$ of $\mathcal{F}$ that map the response d of the information question to the response j of the QCQ. That is, $n_{d,j}$ is the number of l that satisfies $f_l(d) = j$. From the definition, since $h_{l,d} = f_l(d)$, this can be written as:

$$n_{d,j} = \sum_{l=1}^{L} I_j(h_{l,d}). \tag{30}$$

Lemma 2. Let $A(C)$ be the maximum value of the objective function in the following optimization problem for $C > 0$:

$$A(C) = \max_{\alpha}\left\{\sum_{d=0}^{D-1}\sum_{d'=0}^{D-1} n_{d,d'}\alpha_d\alpha_{d'} + \sum_{d=0}^{D-1} n_{d,d}\alpha_d\right\},$$
$$\text{s.t.} \sum_{d=0}^{D-1}\alpha_d = C, \quad \boldsymbol{\alpha} \geq \mathbf{0}. \tag{31}$$

Then, the following inequality holds:

$$Q_{\inf}(\mathcal{F}) \geq 1 - \left\{\sup_C \frac{A(C)}{LC(C+1)}\right\}^L. \tag{32}$$

Proof. From the definition of $Q_{\inf}(\mathcal{F})$, we only need to consider $\sup_{\alpha\geq\mathbf{0}} U_\alpha(\mathcal{F})$. From the inequality of arithmetic and geometric means, it is clear that the following inequality holds:

$$\{U_\alpha(\mathcal{F})\}^{\frac{1}{L}} \leq \frac{\sum_{l=1}^{L}\sum_{d=0}^{D-1}\alpha_d(\alpha_{h_{l,d}}+I_d(h_{l,d}))}{L(\sum_{i=0}^{D-1}\alpha_i)(\sum_{i=0}^{D-1}\alpha_i+1)}. \tag{33}$$

Therefore, it follows that

$$\sup_{\alpha\geq\mathbf{0}} U_\alpha(\mathcal{F}) \leq \left\{\sup_{\alpha\geq\mathbf{0}} \frac{\sum_{l=1}^{L}\sum_{d=0}^{D-1}\alpha_d(\alpha_{h_{l,d}}+I_d(h_{l,d}))}{L(\sum_{i=0}^{D-1}\alpha_i)(\sum_{i=0}^{D-1}\alpha_i+1)}\right\}^L. \tag{34}$$

Next, from the definition of $n_{d,j}$, for any d, the following equation holds:

$$\sum_{l=1}^{L}\alpha_d(\alpha_{h_{l,d}}+I_d(h_{l,d})) = \sum_{d'=0}^{D-1} n_{d,d'}\alpha_d\alpha_{d'} + n_{d,d}\alpha_d. \tag{35}$$

Here, if we set $\sum_{d=0}^{D-1}\alpha_d = C$ for $C > 0$, the denominator of the fraction in Eq. (34) becomes the constant $LC(C+1)$. Under this condition, if we maximize the numerator of Eq. (34) with respect to $\boldsymbol{\alpha}$ using the relation of Eq. (35), it becomes $A(C)$, and the lemma holds. □

Now, given a specific $\mathcal{F}$, we consider finding $A(C)$. If we denote the Lagrangian for Eq. (31) as L_{g}, it can be written using the Lagrange multipliers $\delta_0, \delta_1, \ldots, \delta_{D-1} \geq 0, \lambda$ as follows:

$$L_{\mathrm{g}} = \sum_{d=0}^{D-1}\sum_{d'=0}^{D-1} n_{d,d'}\alpha_d\alpha_{d'} + \sum_{d=0}^{D-1} n_{d,d}\alpha_d + \sum_{d=0}^{D-1}\delta_d\alpha_d - \lambda\left(\sum_{d=0}^{D-1}\alpha_d - C\right). \tag{36}$$

Here, from the conditions of stationary points, $\sum_{i=0}^{D-1}\alpha_i = C$ and the following equations hold for any d:

$$\frac{\partial L_{\mathrm{g}}}{\partial \alpha_d} = \sum_{d'=0}^{D-1}(n_{d,d'}+n_{d',d})\alpha_{d'} + n_{d,d}\alpha_d + \delta_d - \lambda = 0, \tag{37}$$

$$\delta_d\alpha_d = 0. \tag{38}$$

Using these relations, we can find $A(C)$.

Example. Here, we explicitly calculate the right-hand side of Eq. (32) and $Q_{\inf}(\mathcal{F})$ for several patterns of $\mathcal{F}$ in the case of $D = 3$.

[Case 1] $\mathcal{F}$ where f_l for all l is set to p16 in Table 2

In this case, for any $l = 1, 2, \ldots, L$, $h_{l,0} = 1, h_{l,1} = 2$ and $h_{l,2} = 0$. Therefore, $n_{0,1} = n_{1,2} = n_{2,0} = L$, and $n_{d,d'} = 0$ for other patterns of d, d'. Therefore, for any $d = 0, 1, 2$, Eq. (37) becomes:

$$\frac{\partial L_{\mathrm{g}}}{\partial \alpha_d} = L\alpha_{d+1 \bmod 3} + L\alpha_{d+2 \bmod 3} + \delta_d - \lambda = 0. \tag{39}$$

From these equations†, we find that $A(C) = LC^2/3$ when $\alpha_0 = \alpha_1 = \alpha_2 = C/3$. Substituting this relation into the right-hand side of Eq. (32), we obtain:

$$1 - \left\{\sup_C \frac{A(C)}{LC(C+1)}\right\}^L = 1 - \frac{1}{3^L}. \tag{40}$$

Therefore, from Lemmas 1 and 2, we have $Q_{\inf}(\mathcal{F}) = 1 -$

†Note that for each $d = 0, 1, 2$, both the case of $\alpha_d > 0$ and the case of $\alpha_d = 0$ need to be considered.

$1/3^L$.

[Case 2] $\mathcal{F}$ with $L/2$ each of p10 and p26 in Table 2 when L is even

The order of the functions does not affect $Q_{\inf}(\mathcal{F})$, so for $l = 1, 2, \ldots, L/2$, we take f_l of p10 with $h_{l,0} = 1, h_{l,1} = 0, h_{l,2} = 0$, and for $l = L/2+1, L/2+2, \ldots, L$, we take f_l of p26 with $h_{l,0} = 2, h_{l,1} = 2, h_{l,2} = 1$. Therefore, $n_{0,1} = n_{1,0} = n_{2,0} = n_{0,2} = n_{1,2} = n_{2,1} = L/2$, and $n_{d,d'} = 0$ for other patterns of d, d'. In this case, Eq. (37) becomes the same as Eq. (39), we find that $A(C) = LC^2/3$ when $\alpha_0 = \alpha_1 = \alpha_2 = C/3$, as in Case 1. Therefore, $Q_{\inf}(\mathcal{F}) = 1 - 1/3^L$.

[Case 3] $\mathcal{F}$ with $L/3$ each of p8, p12 and p22 in Table 2 when L is a multiple of 3

In this case, since p8, p12, and p22 assign the response of the information questions to different one of QCQs, respectively, $n_{d,d'} = L/3$ for all combinations of d, d'. Substituting these and the conditions $\sum_{d=0}^{D-1} \alpha_d = C$ and Eq. (38) into Eq. (36), we obtain:

$$L_g = \frac{L}{3}\left(\sum_{d=0}^{D-1}\alpha_d\right)\left(\sum_{d'=0}^{D-1}\alpha_{d'}\right) + \frac{L}{3}\left(\sum_{d=0}^{D-1}\alpha_d\right) = \frac{LC(C+1)}{3}. \tag{41}$$

Therefore, since $A(C) = LC(C+1)/3$, from Lemmas 1 and 2, we find that $Q_{\inf}(\mathcal{F}) = 1 - 1/3^L$.

In the above three cases, since $Q_{\inf}(\mathcal{F})$ coincides with the right-hand side value of Eq. (26), $1 - 1/D^L$, we see that these are the best cases in the sense that $Q_{\inf}(\mathcal{F})$ is maximized. On the other hand, we show below that there exist cases that are not necessarily the best.

[Case 4] $\mathcal{F}$ with $L/2$ each of p5 and p24 in Table 2 when L is even

In this case, $n_{0,0} = n_{0,1} = n_{2,1} = n_{2,2} = L/2, n_{1,1} = L$, and $n_{d,d'} = 0$ for other patterns of d, d'. From the conditions of stationary points, we find that $A(C) = LC(C+1)$ when $\alpha_0 = \alpha_2 = 0$ and $\alpha_1 = C$. Substituting this into the right-hand side of Eq. (32), we obtain $Q_{\inf}(\mathcal{F}) \geq 0$. Now, substituting $\alpha_0 = \alpha_2 = 0$ and $\alpha_1 = C$ into Eq. (23), we get $U_\alpha(\mathcal{F}) = 1$. That is, we see that $Q_{\inf}(\mathcal{F}) = 0$ strictly holds in this case.

[Case 5] $\mathcal{F}$ with $L/3$ each of p3, p5 and p24 in Table 2 when L is a multiple of 3

In this case, $n_{0,0} = n_{1,1} = n_{2,2} = 2L/3, n_{0,2} = n_{1,0} = n_{2,1} = L/3$, and $n_{d,d'} = 0$ for other patterns of d, d'. Then, we obtain $A(C) = LC(C+2)/3$ when $\alpha_0 = \alpha_1 = \alpha_2 = C/3$. Substituting this into the right-hand side of Eq. (32), we get $Q_{\inf}(\mathcal{F}) \geq 1 - (2/3)^L$. Now, substituting $\alpha_0 = \alpha_1 = \alpha_2 = C/3$ into Eq. (23) and taking the limit $C \to 0$, we obtain $U_\alpha(\mathcal{F}) \to (2/3)^L$. That is, we see that $Q_{\inf}(\mathcal{F}) = 1-(2/3)^L$ strictly holds in this case.

As in Cases 4 and 5, there exist cases where $Q_{\inf}(\mathcal{F}) < 1 - 1/D^L$. As we have seen in several examples for $D = 3$, we understand that $Q_{\inf}(\mathcal{F})$ differs depending on the choice of $\mathcal{F}$. For more general D and $\mathcal{F}$, the calculation becomes cumbersome due to the increased number of case-by-case analyses at the boundary of the constraints†, but it can be computed in the same way as in this example.

6. Conclusion

In this study, we have examined a framework for detecting poor responses in survey research by incorporating quality control questions. We defined a generalized survey design model that extends traditional methods such as IMC and DQS, and derived the average detection probability of poor responses under a presumed poor response model. Additionally, we provided guidelines for the addition of quality control questions through numerical experiments.

Future challenges include investigating whether the Dirichlet distribution model, which is the poor response model, accurately represents the response behavior of actual poor responders.

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†This refers to the case-by-case analyses of whether $\alpha_d = 0$ or not.

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